This approach is called by us the finite-difference time-domain (FD-TD) solution of Maxwell's curl equations. Our publications through the years [1]–[8] have established that FD-TD can accurately model electromagnetic-wave penetration and scattering interactions with complex metal, dielectric, and biological objects. Our most recent work [9] demonstrates high accuracy ($\pm$ 1 dB over a 40-dB dynamic range) in modeling the scattering properties of a *nine-wavelength three-dimensional* scatterer of complex shape. FD-TD models having *in excess of 10^6 space cells* have been successfully run [10].

We wish to call this to the attention of the authors of the above paper so that in future articles they may place their work in proper perspective, and properly inform their readers of the state-of-the-art.

References

[1] A. Taflove and M E. Brodwin, "Computation of the electromagnetic fields and induced temperatures within a model of the microwave-irradiated human eye," *IEEE Trans. Microwave Theory Tech.*, vol. MTT-23, pp. 888–896, Nov. 1975

[2] A. Taflove and M. E. Brodwin, "Numerical solution of steady-state electromagnetic scattering problems using the time-dependent Maxwell's equations," *IEEE Trans Microwave Theory Tech.*, vol. MTT-23, pp. 623–630, Aug. 1975.

[3] A. Taflove and K. R. Umashankar, "A hybrid moment method/finite-difference time-domain approach to electromagnetic coupling and aperture penetration into complex geometries," invited chapter (no. 14) in *Applications of the Method of Moments to Electromagnetic Fields*, B J. Strain, Ed. Orlando, FL: SCEEE Press, Feb. 1980.

[4] A Taflove, "Application of the finite-difference time-domain method to sinusoidal steady state electromagnetic penetration problems," *IEEE Trans. Electromag. Compat.*, vol. EMC-22, pp. 191–202, Aug. 1980.

[5] A. Taflove and K. R. Umashankar, "Solution of complex electromagnetic penetration and scattering problems in unbounded regions," invited paper in *Computational Methods for Infinite Domain Media-Structure Interaction*, American Society of Mechanical Engineers, AMD vol. 46, pp 83–113, Nov. 1981

[6] A. Taflove and K. R. Umashankar, "A hybrid moment method/finite-difference time-domain approach to electromagnetic coupling and aperture penetration into complex geometries," *IEEE Trans. Antennas Prop.*, vol. AP-30, pp. 617–627, July 1982.

[7] K. R. Umashankar and A. Taflove, "A novel method to analyze electromagnetic scattering of complex objects," *IEEE Trans. Electromag. Compat.*, vol. EMC-24, pp. 397–405, Nov. 1982.

[8] A. Taflove and K. R Umashankar, "Radar cross section of general three-dimensional scatterers," *IEEE Trans. Electromag. Compat.*, vol. EMC-25, pp. 433–440, Nov. 1983.

[9] A. Taflove, K. R. Umashankar, and T G. Jurgens, "Validation of FD-TD modeling of the radar cross section of three-dimensional structures spanning up to 9 wavelengths," presented at IEEE APS/URSI Int. Symp., Boston, MA, June 1984.

[10] A Taflove and K. R. Umashankar, "Review of the state-of-the-art of numerical techniques for analyzing electromagnetic coupling and interaction problems," invited paper to plenary session of the Fourth Nuclear Electromagnetics (NEM) Symposium, Baltimore, MD, July 1984.

Comments on "Application of Boundary-Element Method to Electromagnetic Field Problems"

N. MORITA, SENIOR MEMBER, IEEE

The above paper[1] has explained a general formulation of the boundary-element method (BEM) for analyzing two-dimensional electromagnetic fields, and has presented numerical examples for some boundary shapes to show that the BEM is a very powerful numerical method for solving electromagnetic field problems. It gives accurate results with far fewer nodes than the finite-element method, and can also treat field problems in unbounded regions without any additional complications.

I wonder, however, why such an argument is necessary now. The BEM is not a new method, but just the surface integral equation method which has already been proved to be a very useful method in the areas of electromagnetic and other fields. The literature is extensive on the analysis of electromagnetic field problems by integral equations, on discussions of integral equations themselves, on their discretization methods, etc. References [1] and [2] were probably the first to present a practical and numerical technique using integral equations for electromagnetic field problems. Several good books and review papers describing the use of numerical techniques for integral equations have also been published [3]–[6].

The discretization method in the BEM is explained in detail in the above paper.[1] The method shown is, however, just one of many methods now available. It is the one based on the approximation of unknown functions by means of the triangular subsectional functions, which has been proved to be more effective for some cases than the step function approximation [3], [7], [8], [12]. Of course, there are many other better functions to be used depending upon the problem to be solved.

In Section V of the above paper,[1] an integral equation formulation for scattering from dielectric bodies is presented. However, the problem of scattering from material bodies, such as dielectric and gyrotropic bodies, has been treated extensively in the past literature. Various kinds of integral equations for analyzing these problems are now available [9]–[14]. The set of equations given in the above paper[1] is essentially the same as one of those used in the past [11], [15], [16], and can easily be derived using the integral relation on the incident field. In addition, the equation shown is inferior to ones used in the past since the term involving the incident wave is unnecessarily complex. Furthermore, the problem of erroneous resonant solutions involved in these types of equations is not stated at all. The problem of non-uniqueness, which is often associated with simple surface integral equations, has been discussed by many researchers [17]–[22], [13].

I would like to add that the above paper[1] treats only the two-dimensional problems, even though a lot of numerical results have already been given for three-dimensional electromagnetic field problems. (Some of these can be found in the list of References.)

Finally, I don't think that the whole literature on the integral equation formulation can be neglected by using the "anesthetic" by the name of the "boundary-element method," of which only the label is new.

The author wishes to thank Prof. R. F. Harrington for careful reading of the manuscript.

Reply[2] by Shin Kagami and Ichiro Fukai[3]

The authors of the original paper[1] were aware of the previous works on the electromagnetic field analysis using the boundary integral equation method (BIE) and they consider that the

Manuscript received June 4, 1984
The author is with the Department of Communication Engineering, Faculty of Engineering, Osaka University, Suita, Osaka 565, Japan.
[1] S. Kagami and I. Fukai, *IEEE Trans Microwave Theory Tech.*, vol MTT-32, pp. 455–461, Apr. 1984.

[2] Manuscript received November 2, 1984.
[3] S. Kagami is with the Department of Electrical Engineering, Asahikawa National College of Technology, Asahikawa, 070 Japan.
I Fukai is with the Department of Electrical Engineering, Faculty of Engineering, Hokkaido University, Sapporo, 060 Japan

0018-9480/85/0400-0346$01.00 ©1985 IEEE

boundary-element method (BEM) is based on BIE. However, they thought that the BIE was a common numerical technique already established and confirmed generally in this field, so that they did not refer to it individually.

In the original paper, the authors aimed to emphasize the facility of the application of the BEM, which is an "element method" and whose discretizing technique is like that of the finite-element method (FEM). These facts cause the BEM to become a very powerful numerical method. It is very easy to perform programming for computers. In addition, it adopts simple and general expressions (for example, the equation having a general variable-a single scalar potential), so that the formulation is performed about the scalar Helmholtz's equation, and when actual problems are treated, a proper boundary condition is imposed on the above potential. Moreover, the same program can be used for different cases (for example, for the case of sound problems). Its govering equation is also the scalor Helmholtz's equation, but its boundary condition is different from that of the electromagnetic field problem.

Finally, the authors would like to thank Dr. N. Morita for his remarks and for providing [10], [20], and [21].

References

[1] K. K. Mei and J. Van Baldel, "Low-frequency scattering by rectangular cylinders," *IEEE Trans. Antennas Propagat.*, vol. AP-11, pp. 52–56, Jan 1963.
[2] K. K. Mei and J. Van Bladel, "Scattering by perfectly-conducting rectangular cylinders," *IEEE Trans. Antennas Propagat.*, vol. AP-11, pp. 185–192, Mar. 1963.
[3] R. F. Harrington, *Field Computation by Moment Methods*. New York: MacMillan, 1968.
[4] R. Mittra, Ed., *Computer Techniques for Electromagnetics* Oxford: Pergamon, 1973.
[5] R. Mittra, Ed., *Numerical and Asymptotic Techniques in Electromagnetics* (Topics in Applied Physics, Vol. 3). Berlin: Springer, 1975.
[6] D. S. Jones, "Numerical methods for antenna problems," *Proc Inst. Elec. Eng.*, vol. 121, pp. 573–582, July 1974.
[7] B E. Spielman and R. F. Harrington, "Waveguides of arbitrary cross section by solution of a nonlinear integral eigenvalue equation," *IEEE Trans. Microwave Theory Tech.*, vol. MTT-20, pp. 578–585, Sept. 1972.
[8] N Okamoto, "Computer-aided design of *H*-plane waveguide junctions with full-height ferrites of arbitrary shape," *IEEE Trans. Microwave Theory Tech.*, vol. MTT-27, pp. 315–321, Apr. 1979.
[9] V V Solodukhov and E N. Vasil'ev, "Diffraction of a plane electromagnetic wave by a dielectric cylinder of arbitrary cross section," *Sov. Phys.-Tech. Phys.*, vol. 15, pp. 32–36, July 1970.
[10] N. Morita, "Analysis of scattering by a dielectric rectangular cylinder by means of integral equation formulation," *Trans. Inst. Elec. Commun. Eng Japan*, vol 57-B, pp. 72–80, Oct. 1974.
[11] T.-K. Wu and L. L. Tsai, "Numerical analysis of electromagnetic fields in biological tissues," *Proc. IEEE*, vol. 62, pp. 1167–1168, Aug. 1974.
[12] N. Okamoto, "A method for problems of scattering by gyrotropic composite cylinders," *Trans. Inst. Elec. Commun. Eng. Japan*, vol. 58-B, pp. 269–276, June 1975.
[13] J R. Mautz and R. F. Harrington, "Electromagnetic scattering from a homogeneous material body of revolution," *Arch. Elek. Übertragung.*, vol. 33, pp. 71–80, Feb. 1979.
[14] T-K. Wu and L. L. Tsai, "Scattering from arbitrarily-shaped lossy dielectric bodies of revolution," *Radio Sci.*, vol. 12, pp. 709–718, Sept./Oct. 1977.
[15] K. A. Zaki and A. R. Neureuther, "Scattering from a perfectly conducting surface with a sinusoidal height profile," *IEEE Trans. Antennas Propagat.*, vol. AP-19, pp. 208–214, Mar. 1971
[16] K. K. Mei, "Unimoment method of solving antenna and scattering problems," *IEEE Trans. Antennas Propagat.*, vol. AP-22, pp. 760–766, Nov. 1974.
[17] H. A. Schenck, "Improved integral formulation for acoustic radiation problems," *J. Acoust. Soc. Am.*, vol. 44, pp. 41–58, July 1968
[18] J -C. Bolomey and W. Tabbara, "Numerical aspects on coupling between complementary boundary value problems," *IEEE Trans Antennas Propagat.*, vol. AP-21, pp. 356–363, May 1973.
[19] C. A. Klein and R Mittra, "An application of the 'Condition Number' concept to the solution of scattering problems in the presence of the interior resonant frequencies," *IEEE Trans. Antennas Propagat.*, vol. AP-23, pp. 431–435, May 1975.
[20] N. Morita, "Surface integral representations for electromagnetic scattering from dielectric cylinders," *IEEE Trans. Antennas Propagat.*, vol. AP-26, pp. 261–266, Mar. 1978.
[21] N. Morita, "Resonant solutions involved in the integral equation approach to scattering from conducting and dielectric cylinders," *IEEE Trans. Antennas Propagat.*, vol. AP-27, pp. 869–871, Nov. 1979.
[22] J. R Mautz and R. F. Harrington, "*H*-field, *E*-field, and combined field solutions for bodies of revolution," *Arch. Elek Übertragung.*, vol. 32, pp. 157–164, Apr. 1978.

Comments on "Limitations of the Cubical Block Model of Man in Calculating SAR Distributions"

MARK J. HAGMANN, MEMBER, IEEE

The above paper[1] raised some serious questions regarding the accuracy of three-dimensional block model solutions obtained using a pulse-function basis. While I am in qualitative agreement with about half of the numerical results presented in the paper, I most strongly disagree with most of the interpretations which the authors have made using those results. It is my belief that it is possible to obtain high accuracy with block model solutions if sufficient care is used in their implementation. I have chosen to use a pulse-function basis with block models of man since this appears to allow the model to have much greater detail than is possible with more elaborate bases.

The paper incorrectly stated that I have given "an upper limit on the dimensions of cells for the required accuracy" and inferred that such a limit was satisfied in their solutions. In earlier work with one of the authors (Durney), it was shown that the size of each cell must not be much greater than the reciprocal of the magnitude of the complex propagation vector, but this was presented as a condition that is necessary but not sufficient for convergence [2]. Pulse functions are only appropriate if the electric field is slowly varying over the volume of each cell. The electric field will have sizable variation within some objects even in static solutions. One case in point is the dielectric cube which the authors unfortunately chose to use as an example.

The solution for a 27-cell block model of a dielectric cube, as presented in the article, is very far from convergence. While an exact solution is not available for the dielectric cube, it is generally known that the electric field is highly heterogeneous near the corners and edges. While I have not obtained a solution for a cube having the exact parameters used by the authors, the results of earlier studies [3], [4], as well as recent work using as many as 2744 cells, suggests that the fields near corners and edges are sufficiently intense that the true average SAR would be several times greater than that calculated for a 27-cell block model. I am not surprised that subdividing the cell at the center of the cube had little effect since it is well known that at low frequencies the electric field at the center of a cube is the same as that at the center of a sphere, and the solution for a small number of cells is more like that for a sphere than a cube. I am also not surprised that subdividing a cell at a corner or edge of the cube caused a significant change in the SAR since these are regions where the 27-cell solution has the greatest error.

Manuscript received August 27, 1984.
The author is with the National Institutes of Health, Bethesda, MD 20205.
[1] H. Massoudi, C. H. Durney, and M F. Iskander, *IEEE Trans. Microwave Theory Tech.*, vol. MTT-32, pp 746–752, Aug 1984.